A- Special Functions

(1) Find the following

(1) $\Gamma(3) + \Gamma(4)$ (2) $\Gamma(5).\Gamma(3)$ (3) $\Gamma(0.5).\Gamma(1.5)$

(4) $\Gamma(0.5).\Gamma(-0.5)$ (5) $\Gamma(1.5).\Gamma(-0.5)$ (6) $B(1, 3) + B(3, 2)$

(7) $\Gamma(-0.5).B(2 , 0.5)$ (8) $\Gamma(5) + B(3, 4)$ (9) $B(\Gamma(2), \Gamma(3))$

(10) $B(\frac{1}{4},\frac{7}{4}) + B(\frac{1}{3},\frac{5}{3})$ (11) $\frac{\Gamma(3.5)}{B(3,4)} + B(\frac{1}{3},\frac{5}{3})$ (12) $\frac{\Gamma(2.5)}{B(4,2)}.B(\frac{1}{4},\frac{7}{4})$

(2) Find the following integrals

(i) $\int_0^\infty e^{-x^4}dx$ (ii) $\int_0^\infty xe^{-x^3}dx$ (iii) $\int_0^1 x^3(-\ln x)^4dx$

(iv) $\int_0^\infty x^5e^{-2x^2}dx$ (v) $\int_2^\infty e^{4x-x^2}dx$ (vi) $\int_3^\infty e^{-x^2+6x}dx$

(vii) $\int_0^1 \sqrt[3]{1-x^3}dx$ (viii) $\int_0^3 x.\sqrt[3]{27-x^3}dx$ (ix) $\int_0^{\frac{\pi}{2}} \sec x\sqrt{\cot x}dx$

(x) $\int_3^4 (x-3)^4.\sqrt{4-x}dx$ (xi) $\int_0^{\frac{\pi}{2}} \sin x\sqrt{\tan x}dx$ (xii) $\int_0^2 x.\sqrt[4]{16-x^4}dx$

B- Laplace Transformations

(1)Find F(s) of the following functions:

(1) $f(t) = 2t^3 - 5t^2 + 3$

(2) $f(t) = t + (t+2)^2$

(3) $f(t) = t^3 (t+3)^2$

(4) $f(t) = (t^2+1)^3$

(5) $f(t) = t.\sqrt{t} - 4$

(6) $f(t) = t^2 + \sqrt[3]{t} + 4$

(7) $f(t) = 4\sin 3t + 5e^{2t}$

(8) $f(t) = 2\sinh 4t + \cos 2t$

(9) $f(t) = (4 - e^{-t})^2$

(10) $f(t) = (1 - e^{t})(2 + e^{-t})$

(11) $f(t) = e^{t+3}(e^{2t} + 3e^{-t})$

(12) $f(t) = 4 + 3^{2t}$

(13) $f(t) = (\sin t - \cos t)^2$

(14) $f(t) = (3 - 2\sin 2t)^2$

(15) $f(t) = (1 - \cos 3t)(2 + \cos 3t)$

(16) $f(t) = \sin 3t \cos 2t$

(17) $f(t) = \sin 4t \sin 3t$

(18) $f(t) = \cos 2t \cos 5t$

(19) $f(t) = \sin^4 t$

(20) $f(t) = \cos^4 t$

(21) $f(t) = \sin 2t \cos^2 3t$

(22) $f(t) = \sin^3 2t$

(23) $f(t) = \sin(2t + \pi/3)$

(24) $f(t) = \cosh^2 3t$

(25) $f(t) = \sinh 2t \cosh 3t$

(26) $f(t) = \cos t . \sinh 2t$

(27) $f(t) = t^2 \cosh 3t$

(28) $f(t) = \sin(t - \pi), \quad t > \pi.$

(29) $f(t) = t^2 \sin 3t . e^{4t}$

(30) $f(t) = \dfrac{e^{-2t} - e^{-3t}}{t}$

(31) $f(t) = \dfrac{\sin 2t}{t}$

(32) $f(t) = \dfrac{\cos 3t - \cos 2t}{t}$

(33) $f(t) = \int_0^t \dfrac{\sin t}{t} dt$

(34) $f(t) = t.\int_0^t \dfrac{\sin t}{t} dt$

(35) $f(t) = \int_0^t \dfrac{e^{-2t} - e^{3t}}{t} dt$

(36) $f(t) = e^{2t} \int_0^t \dfrac{e^{-2t} - e^{3t}}{t} dt$

(37) $f(t) = \partial_0(t) + \partial_3(t)$

(38) $f(t) = 2\partial_3(t) - 3\partial_4(t)$

(39) $f(t) = \partial_0(t) \cos t + t.\partial_3(t)$

(37) $f(t) = t + \partial_3(t).e^{2t}$

(2)Find the inverse Laplace transform of the following:

(1) $F(s) = \frac{3}{s} - \frac{1}{s^2+1}$

(2) $F(s) = \frac{3}{s-2} - \frac{1}{s^2+3}$

(3) $F(s) = \frac{1}{s-2} - \frac{2s}{s^2-9}$

(4) $F(s) = \frac{1}{s^4} + \frac{2}{(s+1)^2+1}$

(5) $F(s) = \frac{s}{s^2+2s-3}$

(6) $F(s) = \frac{3}{s^2-4s+3}$

(7) $F(s) = \frac{s}{s^2-6s+9}$

(8) $F(s) = \frac{s+3}{s^2-4s+4}$

(9) $F(s) = \frac{2}{s^2-6s+9}$

(10) $F(s) = \frac{3}{s^2-4s+4}$

(11) $F(s) = \frac{s}{s^2+2s+10}$

(12) $F(s) = \frac{s+1}{s^2+6s+5}$

(13) $F(s) = \frac{s-1}{s^2+2s+2}$

(14) $F(s) = \frac{2}{s^2-2s+5}$

(15) $F(s) = \frac{1}{(2s-3)^3}$

(16) $F(s) = \frac{s}{s^2+16} e^{-2s}$

(17) $F(s) = \ln(\frac{s+2}{s+1})$

(18) $F(s) = \ln \frac{s^2+1}{(s-1)^2}$

(19) $F(s) = \tan^{-1}(s+1)$

(20) $F(s) = \frac{1}{s} \ln \frac{s+3}{s+5}$

(21) $F(s) = \frac{1}{s(s^2+1)}$

(22) $F(s) = \frac{4}{s^3(s-2)}$

(23) $F(s) = 3 + \frac{s^2}{s^2+1}$

(24) $F(s) = e^{-2s} + \frac{s+2}{s-3}$

(3)Solve the following differential equations:

(1) $y` - 2y = 0, \; y(0) = 2$ Ans: $y = 2e^{2t}$

(2) $y`` + 4y` - 5y = 0, \; y(0) = 1, \; y`(0) = 0$ Ans: $y = (\cosh 3t + \frac{2}{3}\sinh 3t)e^{-2t}$

(3) $y`` - 4y = 0, \; y(0) = -1, \; y`(0) = 1$ Ans: $y = \frac{1}{2}\sinh 2t - \cosh 2t$

(4) $y`` + 5y` + 6y = 0, \; y(0) = y`(0) = 1$ Ans: $y = 4e^{-2t} - 3e^{-3t}$

(5) $y'' + 2y' + 5y = 0,\ y(0) = y'(0) = 1$ Ans: $y = (\cos 2t + \sin 2t)\, e^{-t}$

(6) $y' + 3y = t + 1,\ y(0) = 0$ Ans: $y = -\frac{2}{9}e^{-3t} + \frac{1}{3}t + \frac{2}{9}$

(7) $y'' + 4y' + 3y = e^{-t},\ y(0) = y'(0) = 1$ Ans: $y = -\frac{3}{4}e^{-3t} + \frac{7}{4}e^{-t} + \frac{1}{2}t\, e^{-t}$

(8) $y'' - 3y' + 2y = 6e^{2t},\ y(0) = y'(0) = 3$ Ans: $y = 9e^{t} - 6e^{2t} + 6te^{2t}$

C- Fourier Series

(1)Find the Fourier series of the following functions:

(a) $f(x) = 4 - x^2, \quad -2 \le x \le 2, \quad f(x+4) = f(x)$

(b) $f(x) = x^2, \quad -2 \le x \le 2, \quad f(x+4) = f(x)$

(c) $f(x) = x + 1, \quad -1 \le x \le 1, \quad f(x+2) = f(x)$

(d) $f(x) = x \sin x, \quad -\pi \le x \le \pi \quad f(x+2\pi) = f(x)$

(e) $f(x) = |x|, \quad -3 \le x \le 3, \quad f(x+6) = f(x)$

(f) $f(x) = |\sin x|, \quad -\pi \le x \le \pi \quad f(x+2\pi) = f(x)$

(g) $f(x) = |x| - 1, \quad -1 \le x \le 1, \quad f(x+2) = f(x)$

(h) $f(x) = \begin{cases} -1, & -1 \le x \le 0 \\ 1, & 0 < x \le 1 \end{cases} \quad f(x+2) = f(x).$

(i) $f(x) = \begin{cases} 1 + x, & -1 \le x \le 0 \\ 1 - x, & 0 < x \le 1 \end{cases} \quad f(x+2) = f(x).$

(j) $f(x) = \begin{cases} 0, & -2 \le x \le 0 \\ x^2, & 0 < x \le 2 \end{cases} \quad f(x+4) = f(x)$

(k) $f(x) = \begin{cases} 0, & -2 \le x \le 0 \\ 1, & 0 < x \le 2 \end{cases} \quad f(x+4) = f(x)$

(l) $f(x)=\begin{cases}-x, & -2\le x\le 0\\ o, & 0<x\le 2\end{cases}$ $\qquad f(x+4)=f(x)$

(m) $f(x)=\begin{cases}-2-x, & -2\le x\le 0\\ 2-x, & 0<x\le 2\end{cases}$ $\qquad f(x+4)=f(x)$

(n) $f(x)=\begin{cases}0, & -\pi\le x\le 0\\ \sin x, & 0<x\le \pi\end{cases}$ $\qquad f(x+2\pi)=f(x)$

(o) $f(x)=\sin^5 x$, x in $[0, 2\pi]$, $\qquad f(x+2\pi)=f(x)$

(2) If $f(x) = \sin x$, x in $[0, \pi]$, $f(x + 2\pi) = f(x)$

Find: (a) Fourier sine series (b) Fourier cosine series

(3) If $f(x) = x + 1$, x in $[0, 1]$, $f(x + 2) = f(x)$

Find: (a) Fourier sine series (b) Fourier cosine series

(4) If $f(x)=x^2,\quad 0\le x\le 2,\qquad f(x+4)=f(x)$

Find: (a) Fourier sine series (b) Fourier cosine series

(5) $f(x)=\begin{cases}x, & 0\le x\le 1\\ 2-x, & 1<x\le 2\end{cases}$ $\qquad f(x+4)=f(x)$

Find: (a) Fourier sine series (b) Fourier cosine series

(6) Find the Fourier series of the following functions, x in$[-\pi, \pi]$:

(a) $f(x)=\cos^2 x$ (b) $f(x)=\cos^4 x$ (c) $f(x)=\sin^3 x$

(7) Using the Fourier series of the function of problem(1.d) to find the sum of the

series: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)(2n+1)}=\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}...$

(8) Using the Fourier series of the function of problem(1.e) to find the sum of the

series: $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=1+\frac{1}{3^2}+\frac{1}{5^2}+...$

(9) Using the Fourier series of the function of problem(1.k) to find the sum of the

series: $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)}=1-\frac{1}{3}+\frac{1}{5}...$

(10) Find the Fourier integrals of the following functions:

(a) $f(x)=\begin{cases} x-1, & |x|\le 2 \\ 0, & |x|>2 \end{cases}$ (b) $f(x)=\begin{cases} x, & |x|\le 3 \\ 0, & |x|>3 \end{cases}$

(c) $f(x)=\begin{cases} |x|, & |x|\le 4 \\ 0, & |x|>4 \end{cases}$ (d) $f(x)=\begin{cases} \sin x, & |x|\le \pi \\ 0, & |x|>\pi \end{cases}$

(e) $f(x)=\begin{cases} 1-x, & |x|\le 1 \\ 0, & |x|>1 \end{cases}$ (f) $f(x)=\begin{cases} x+1, & |x|\le 1 \\ 0, & |x|>1 \end{cases}$

(g) $f(x)=\begin{cases} 2x, & |x|\le 3 \\ 0, & |x|>3 \end{cases}$ (h) $f(x)=\begin{cases} \cos x, & |x|\le \pi \\ 0, & |x|>\pi \end{cases}$

(11) If $f(x)=\begin{cases} x-2, & 0\le x\le 2 \\ 0, & x>2 \end{cases}$

Find its: (a) Fourier sine transform (b) Fourier cosine transform

(12) If $f(x)=\begin{cases} x, & 0\le x\le 3 \\ 0, & x>3 \end{cases}$

Find its: (a) Fourier sine transform (b) Fourier cosine transform

(13) Find the Fourier sine and Fourier cosine transform of the function:

$f(x)=e^{-x}, x\ge 0$. Then show that:

(a) $\int_0^\infty \frac{x\sin x}{1+x^2}dx=\frac{\pi}{2e}$ (b) $\int_0^\infty \frac{dx}{(1+x^2)^2}=\frac{\pi}{4}$ (c) $\int_0^\infty \frac{x^2dx}{(1+x^2)^2}=\frac{\pi}{4}$

(14) From the problem (1.e), show that: $\int_0^\infty \frac{x\sin x-\sin x}{x^3}\cos\frac{x}{2}dx=\frac{3\pi}{16}$

(15) From the problem (3), show that: $\int_0^\infty \left(\frac{1-\cos x}{x}\right)^2 dx=\frac{\pi}{2}$

D- Partial Differential Equations

(1)Solve the following P.D. equations:

(i) $u_x + 3u_y - 2u = 0$

(ii) $2u_x + 3u_y - 4u = 0$

(iii) $u_x - u_y = 4u$

(iv) $2u_x + u_y - 3x = 0$

(v) $3u_x + 4u_y - 5u = 10y$

(vi) $4u_x + 3u_y - 10u = 5$

(2)Find the solution of each of the following P.D.E:

(i) $2u_x + 3u_y - u = 0$, $u(x, 0) = 4$

(ii) $3u_x - u_y + u = 0$, $u(0, y) = 3e^{5y}$

(iii) $2u_x + u_y + u = 0$, $u(x, 0) = 4e^{-2x}$

(3) Classify and solve the following P.D.E:

(i) $u_{xx} - 5u_{xy} + 6u_{yy} = 0$

(ii) $u_{xx} - 5u_{xy} + u_{yy} = e^{2x+3y}$

(iii) $u_{xx} - 2u_{xy} - 3u_{yy} = 0$

(iv) $u_{xx} - 5u_{xy} + 6u_{yy} = e^{3x+y}$

(v) $u_{xx} - 4u_{xy} + 4u_{yy} = xy^2$

(vi) $u_{xx} - 4u_{xy} = \cos(2x + 3y)$

(vii) $u_{xx} + 3u_{yy} = 3x + y^2$

(viii) $u_{xx} - 4u_{xy} = \sin(4x + y)$

(4)Solve the following P.D.E:

(i) $u_{xx} - u_{xy} - 2u_{yy} + 2u_x + u_y - 5u = 0$

(ii) $u_{xx} + 3u_{xy} + 2u_{yy} + u_x - u_y + 6u = 0$

(iii) $u_{xx} - 4u_{xy} + u_x - 4u_y - u = 0$

(iv) $u_{xx} - 9u_{yy} - 2u_y + 4u = 0$

(5)Solve the following wave equations:

(i) $4u_{xx} = u_{tt}$, $0 < x < 3$

B.C: $u(0, t) = u(3, t) = 0$

I.C: $u(x, 0) = 5$, $u_t(x, 0) = x$

(ii) $9u_{xx} = u_{tt}$, $0 < x < 2$

B.C: $u(0, t) = u(2, t) = 0$

I.C: $u(x, 0) = x - 1$, $u_t(x, 0) = 4x$

(iii) $25u_{xx} = u_{tt}$, $0 < x < 3$

B.C: $u(0, t) = u(3, t) = 0$

I.C: $u(x, 0) = x + 1$, $u_t(x, 0) = x$

(iv) $4u_{xx} = 9u_{tt}$, $0 < x < 3$

B.C: $u(0, t) = u(3, t) = 0$

I.C: $u(x, 0) = x$, $u_t(x, 0) = x$